

Cohomologies of superalgebra of pointwise superproduct

S.E.Konstein^{*} and I.V.Tyutin^{†‡}

I.E.Tamm Department of Theoretical Physics,

P. N. Lebedev Physical Institute,

119991, Leninsky Prospect 53, Moscow, Russia.

Abstract

We consider associative superalgebra realized on the smooth Grassmann-valued functions with compact supports in \mathbb{R}^n . The lower Hochschild cohomologies of this superalgebra are found.

1 Introduction

The hope to construct the quantum mechanics on nontrivial manifolds is connected with geometrical or deformation quantization [1] - [4]. The functions on the phase space are associated with the operators, and the product and the commutator of the operators are described by associative \ast -product and \ast -commutator of the functions. These \ast -product and \ast -commutator are the deformations of usual product and of usual Poisson bracket.

The gauge theories on the noncommutative spaces, the so-called noncommutative gauge theories, are formulated in terms of the \ast -product (see [5] and [6] and references therein).

The structure and the properties of the \ast -product on the usual (even) manifolds are investigated in details [2], [3] and [4]. On the other hand, the \ast -product on supermanifolds is not investigated sufficiently.

It is easy to extend formally the noncommutative product proposed in [7] to the supercase [8], however, there is a problem of the uniqueness of the \ast -product (the uniqueness of the "pointwise" product deformation). In [9], the general form of the associative \ast -product, treated as a deformation of the "pointwise" product, on the Grassman algebra of a finite number of generators, is found and uniqueness is proved. It is interesting to solve analogous problem for functions of even and odd variables.

Each \ast -product generates \ast -commutators, but converse is not true (see discussion in [10]). It seems that non Moyal deformations of Poisson superbracket found in [11] are not generated by \ast -product.

^{*}E-mail: konstein@lpi.ru

[†]E-mail: tyutin@lpi.ru

[‡]This work was supported by the RFBR (grant No. 05-02-17217), and by the grant LSS-4401.2006.2.

The problem of finding $*$ -products is connected with the problem of calculating Hochschild cohomologies. The Hochschild cohomologies of the algebra of smooth functions on \mathbb{R}^n are calculated in [12].

Below we calculate lower Hochschild cohomologies (up to 2-nd) of the algebra of smooth Grassmann valued functions with compact supports on \mathbb{R}^n . Because we need these cohomologies for finding $*$ -product, which is even 2-form, we consider only even cocycles in this paper.

2 Notation

Let \mathbb{K} be either \mathbb{R} or \mathbb{C} . We denote by $\mathcal{D}(\mathbb{R}^n)$ the space of smooth \mathbb{K} -valued functions with compact support on \mathbb{R}^n . This space is endowed with its standard topology: by definition, a sequence $\varphi_k \in \mathcal{D}(\mathbb{R}^n)$ converges to $\varphi \in \mathcal{D}(\mathbb{R}^n)$ if the supports of all φ_k are contained in a fixed compact set, and $\partial^\lambda \varphi_k$ converge uniformly to $\partial^\lambda \varphi$ for every multi-index λ . We set

$$\mathbf{D}_{n_+}^{n_-} = \mathcal{D}(\mathbb{R}^{n_+}) \otimes \mathbb{G}^{n_-}, \quad \mathbf{D}_{n_+}'^{n_-} = \mathcal{D}'(\mathbb{R}^{n_+}) \otimes \mathbb{G}^{n_-},$$

where \mathbb{G}^{n_-} is the Grassmann algebra with n_- generators and $\mathcal{D}'(\mathbb{R}^{n_+})$ is the space of continuous linear functionals on $\mathcal{D}(\mathbb{R}^{n_+})$. The generators of the Grassmann algebra (resp., the coordinates of the space \mathbb{R}^{n_+}) are denoted by ξ^α , $\alpha = 1, \dots, n_-$ (resp., x^i , $i = 1, \dots, n_+$). We shall also use collective variables z^A which are equal to x^A for $A = 1, \dots, n_+$ and are equal to ξ^{A-n_+} for $A = n_+ + 1, \dots, n_+ + n_-$. The spaces $\mathbf{D}_{n_+}^{n_-}$ possess a natural grading which is determined by that of the Grassmann algebra. The parity of an element f of these spaces is denoted by $\varepsilon(f)$. We also set $\varepsilon_A = 0$ for $A = 1, \dots, n_+$ and $\varepsilon_A = 1$ for $A = n_+ + 1, \dots, n_+ + n_-$.

The integral on $\mathbf{D}_{n_+}^{n_-}$ is defined by the relation $\int dz f(z) = \int_{\mathbb{R}^{n_+}} dx \int d\xi f(z)$, where the integral on the Grassmann algebra is normed by the condition $\int d\xi \xi^1 \dots \xi^{n_-} = 1$. We identify \mathbb{G}^{n_-} with its dual space \mathbb{G}^{n_-} setting $f(g) = \int d\xi f(\xi)g(\xi)$, $f, g \in \mathbb{G}^{n_-}$. Correspondingly, the space $\mathbf{D}_{n_+}'^{n_-}$ of continuous linear functionals on $\mathbf{D}_{n_+}^{n_-}$ is identified with the space $\mathcal{D}'(\mathbb{R}^{n_+}) \otimes \mathbb{G}^{n_-}$. As a rule, the value $m(f)$ of a functional $m \in \mathbf{D}_{n_+}'^{n_-}$ on a test function $f \in \mathbf{D}_{n_+}^{n_-}$ will be written in the “integral” form: $m(f) = \int dz m(z)f(z)$.

Let L be a superalgebra of functions $f(z) \in \mathbf{D}_{n_+}^{n_-}$ with the usual “pointwise” product.

Consider an algebra $\mathcal{A} = \sum_{\oplus} \mathcal{A}_k$, where \mathcal{A}_k is the vector space of even k -linear separately continuous forms $\Phi_k(z|f_1, \dots, f_k) \in \mathcal{A}_k$, $k = 0, 1, 2, \dots$ taking value in $\mathbf{D}_{n_+}^{n_-}$, $\varepsilon(\Phi_k(z|f_1, \dots, f_k)) = \sum_{i=1}^k \varepsilon(f_i)$, $\mathcal{A}_0 \subset \mathbf{D}_{n_+}^{n_-}$, $\Phi_0(z) = \phi(z) \in \mathbf{D}_{n_+}^{n_-}$, $\varepsilon_{\Phi_0} = \varepsilon(\phi) = 0$, with (noncommutative) associative product

$$(\Phi_k \diamond \Psi_p)(z|f_1, \dots, f_{k+p}) = \Phi_k(z|f_1, \dots, f_k) \Psi_p(z|f_{k+1}, \dots, f_{k+p}).$$

This algebra has a natural grading g : $g(\Phi_k) = k$, $g(\mathcal{A}_k) = k$, and the Hochschild differential d_H : $\mathcal{A}_k \rightarrow \mathcal{A}_{k+1}$, $g(d_H) = 1$, which acts by the rule:

$$\begin{aligned} d_k \Phi_k(z|f_1, \dots, f_{k+1}) &= f_1(z) \Phi_k(z|f_2, \dots, f_{k+1}) \\ &+ \sum_{i=1}^k (-1)^i \Phi_k(z|f_1, \dots, f_{i-1}, f_i f_{i+1}, f_{i+2}, \dots, f_{k+1}) \\ &+ (-1)^{k+1} \Phi_k(z|f_1, \dots, f_k) f_{k+1}(z), \end{aligned} \tag{2.1}$$

Here $d_k \stackrel{def}{=} d_H|_{\mathcal{A}_k}$. Differential d_H has the following evident properties

$$\begin{aligned} d_0 \Phi_0(z|f) &= f(z)\phi(z) - \phi(z)f(z) = 0, \\ d_{k+p}(\Phi_k \Psi_p) &= (d_k \Phi_k) \Psi_p + (-1)^{g(\Phi_k)} \Phi_k d_p \Psi_p, \\ d_{k+1} d_k &= 0, \quad k = 0, 1, \dots \end{aligned} \tag{2.2}$$

Zeroth, first and second Hochschild cohomologies are found in successive sections.

3 H^0

The cohomological equation

$$d_0 \Phi_0(z|f) = 0$$

is satisfied identically for any forms $\Phi_0(z)$, such that we have $H^0 = \mathcal{A}_0$.

4 H^1

In this case the cohomological equation has the form

$$f(z)\Phi_1(z|g) - \Phi_1(z|fg) + \Phi_1(z|f)g(z) = 0. \tag{4.1}$$

Let¹

$$[z \cup \text{supp}(g)] \cap \text{supp}(f) = \emptyset. \tag{4.2}$$

We obtain from Eq. (4.1)

$$\hat{\Phi}_1(z|f) = 0,$$

from what it follows²

$$\Phi_1(z|f) = \sum_{q=0}^Q m(z)^{(A)_q} (\partial_A)^q f(z).$$

Here $\hat{\Phi}_1(z|f)$ means the restriction of the linear form $\Phi_1(z|f)$ on the domain (4.2) or, equivalently, on the domain $z \cap \text{supp}(f) = \emptyset$,

Analogously to method used in [14], [13] let us choose $f(z) = e^{zp}$, $g(z) = e^{zk}$ in some neighborhood of x . We obtain from Eq. (4.1)

$$F(z|k) - F(z|p+k) + F(z|p) = 0, \tag{4.3}$$

where

$$F(z|p) = \sum_{q=0}^Q m(z)^{(A)_q} (p_A)^q.$$

Eq. (4.3) gives

$$F(z|p) = m(z)^A p_A$$

¹Let $z = (x, \xi)$. Here and below we use the notation $z \cup U$ ($z \cap U$) instead of $V_z \cup U$ ($V_z \cap U$) where V_z is some neighborhood of x .

²details can be found in [13].

or

$$\Phi_1(z|f) = m(z)^A \partial_A f(z).$$

It is obvious that the cohomologies with different $m(z)^A$ are independent.

5 H^2

The cohomological equation has the form

$$f(z)\Phi_2(z|g, h) - \Phi_2(z|fg, h) + \Phi_2(z|f, gh) - \Phi_2(z|f, g)h(z) = 0. \quad (5.1)$$

Let $\mathcal{L}_k \subset \mathcal{A}_k$ be the spaces of local forms. The local form is such $\Phi_k \in \mathcal{A}_k$ that if $z \cap \text{supp}(f_i) = \emptyset$ for some $1 \leq i \leq k$ then $\Phi_k(z|f_1, \dots, f_k) = 0$.

5.1 Nonlocal part of cocycle

Let

$$[z \cup \text{supp}(h)] \cap \text{supp}(f) = [z \cup \text{supp}(h)] \cap \text{supp}(g) = \text{supp}(f) \cap \text{supp}(g) = \emptyset. \quad (5.2)$$

We obtain from Eq. (5.1)

$$\hat{\Phi}_2(z|f, g) = 0,$$

which yields

$$\Phi_2(z|f, g) = \sum_{q=0}^Q \{m_1^{(A)q}(z|f)(\partial_A)^q g(z) + [(\partial_A)^q f(z)]m_2^{(A)q}(z|g) + m_3^{(A)q}(z|f(\partial_A)^q g)\}. \quad (5.3)$$

Here notation $\hat{\Phi}_2(z|f, g)$ is used for nondiagonal part of $\Phi_2(z|f, g)$, i.e. for the restriction of Φ on the domain (5.2) or on the domain $z \cap \text{supp}(f) = z \cap \text{supp}(g) = \text{supp}(f) \cap \text{supp}(g) = \emptyset$.

Let us note that all the forms $m_a^{(A)q}$, and the form $m_1^{(A)0}$ particularly, are globally defined distributions.

Consider the domain

$$[z \cup \text{supp}(f) \cup \text{supp}(h)] \cap \text{supp}(g) = \emptyset. \quad (5.4)$$

We obtain from Eq. (5.1) the equation for the restriction of the form Φ on the domain (5.4)

$$f(z)\hat{\Phi}_2(z|g, h) - \hat{\Phi}_2(z|f, g)h(z) = 0. \quad (5.5)$$

Substituting representation (5.3) in Eq. (5.5), we find

$$\sum_{q=0}^Q \{f(z)\hat{m}_1^{(A)q}(z|g)(\partial_A)^q h(z) - [(\partial_A)^q f(z)]\hat{m}_2^{(A)q}(z|g)h(z)\} = 0,$$

from what it follows

$$\begin{aligned} m_1^{(A)q}(z|g) &\in \mathcal{L}_1, \quad m_2^{(A)q}(z|g) \in \mathcal{L}_1, \quad q \geq 1, \\ m_2^{(A)0}(z|g) - m_1^{(A)0}(z|g) &\in \mathcal{L}_1. \end{aligned}$$

So, the form $\Phi_2(z|f, g)$ can be represented in the form

$$\begin{aligned}\Phi_2(z|f, g) &= \left[\sum_{q=0}^Q m_3^{(A)q}(z|f(\partial_A)^q g) + m_1^{(A)0}(z|fg) \right] + \\ &+ [f(z)m_1^{(A)0}(z|g) - m_1^{(A)0}(z|fg) + m_1^{(A)0}(z|f)g(z)] + \Phi_{\text{loc}}(z|f, g), \\ \Phi_{\text{loc}}(z|f, g) &\in \mathcal{L}_2\end{aligned}$$

or, redefining $m_3^{(A)0}(z|f)$,

$$\begin{aligned}\Phi_2(z|f, g) &= \Phi_{2|3}(z|f, g) + d_1 \Phi_{1|1}(z|f, g) + \Phi_{\text{loc}}(z|f, g), \quad \Phi_{1|1}(z|f) = m_1^{(A)0}(z|f), \quad \varepsilon_{\Phi_{1|1}} = 0, \\ \Phi_{2|3}(z|f, g) &= \sum_{q=0}^Q m_3^{(A)q}(z|f(\partial_A)^q g).\end{aligned}\tag{5.7}$$

To specify $\Phi_{2|3}$, consider the domain

$$z \cap [\text{supp}(f) \cup \text{supp}(g) \cup \text{supp}(h)] = \emptyset.\tag{5.8}$$

Using representation (5.6), we obtain from Eq. (5.1)

$$\hat{\Phi}_{2|3}(z|fg, h) - \hat{\Phi}_{2|3}(z|f, gh) = 0.\tag{5.9}$$

Substituting representation (5.7) in Eq. (5.9), we find

$$\sum_{q=0}^Q \hat{m}_3^{(A)q}(z|\{fg(\partial_A)^q h - f(\partial_A)^q(gh)\}) = 0,$$

Choosing $g(z) = e^{-zp}$, $h(z) = e^{zp}$ on $\text{supp}(f)$, we obtain

$$\hat{F}(z|f; p) - \hat{F}(z|f; 0) = 0, \quad F(z|f; p) = \sum_{q=0}^Q \hat{m}_3^{(A)q}(z|f)(p_A)^q,$$

and then

$$\hat{m}_3^{(A)q}(z|f) = 0 \Rightarrow m_3^{(A)q}(z|f) \in \mathcal{L}_1, \quad q \geq 1.$$

So, the form $\Phi_2(z|f, g)$ can be represented in the form

$$\Phi_2(z|f, g) = m(z|fg) + d_1 \Phi_{1|1}(z|f, g) + \Phi_{\text{loc}}(z|f, g), \quad m(z|f) = m_3^{(A)0}(z|f).\tag{5.10}$$

At last, consider the domain

$$[z \cup \text{supp}(h)] \cap [\text{supp}(f) \cup \text{supp}(g)] = \emptyset.\tag{5.11}$$

Using representation (5.10), we obtain from Eq. (5.1)

$$m(z|fgh) + m(z|fg)h(z) = 0$$

and

$$m(z|f) = 0.$$

Finally, we have obtained

$$\Phi_2(z|f, g) = \Phi_{2|\text{loc}}(z|f, g) + d_1 \Phi_{1|1}(z|f, g), \quad (5.12)$$

$$\Phi_{2|\text{loc}}(z|f, g) = \sum_{k,l=0}^Q f(z) (\overleftarrow{\partial}_A)^k m^{(A)_k|(B)_l}(z) (\partial_B)^l g(z), \quad \varepsilon_{m^{(A)_k|(B)_l}} = \varepsilon_{A_1} + \cdots + \varepsilon_{B_l},$$

$$(\partial_A)^k = \partial_{A_k} \cdots \partial_{A_1}, \quad (\overleftarrow{p}_A)^k = p_{A_k} \cdots p_{A_1}, \quad (\overleftarrow{\partial}_A)^k = \overleftarrow{\partial}_{A_1} \cdots \overleftarrow{\partial}_{A_k}, \quad (p_A)^k = p_{A_1} \cdots p_{A_k},$$

$$m^{(A)_k} = m^{A_1 \cdots A_k}, \quad m^{\cdots A_i A_{i+1} \cdots} = (-1)^{\varepsilon_{A_i} \varepsilon_{A_{i+1}}} m^{\cdots A_{i+1} A_i \cdots},$$

where all the coefficients $m^{(A)_k|(B)_l}(z)$ and the form $\Phi_{1|1}(z|f)$ are defined globally.

One can see that nonlocal parts of cocycles are exact forms.

5.2 Local cocycles

For local form, the cohomological equation (5.1) takes the form

$$\sum_{k,l=0}^Q \left(f(z) [g(z) (\overleftarrow{\partial}_A)^k] m^{(A)_k|(B)_l}(z) (\partial_B)^l h(z) - [f(z) g(z)] (\overleftarrow{\partial}_A)^k m^{(A)_k|(B)_l}(z) (\partial_B)^l h(z) + \right.$$

$$\left. + f(z) (\overleftarrow{\partial}_A)^k m^{(A)_k|(B)_l}(z) (\partial_B)^l [g(z) h(z)] - f(z) (\overleftarrow{\partial}_A)^k m^{(A)_k|(B)_l}(z) [(\partial_B)^l g(z)] h(z) = 0 \right). \quad (5.13)$$

Let $f(z) = e^{pz}$, $g(z) = e^{qz}$, $h(z) = e^{rz}$ in some neighborhood of x . Then the cohomological equation transforms to the form

$$F(z|q, \tilde{r}) - F(z|p+q, \tilde{r}) + F(z|p, \tilde{q} + \tilde{r}) - F(z|p, \tilde{q}) = 0, \quad (5.14)$$

$$F(z|p, q) = \sum_{k,l=0}^Q (p_A)^k m^{(A)_k|(B)_l}(z) (q_B)^l, \quad \tilde{p}_A = (-1)^{\varepsilon_A} p_A.$$

Let us apply the operator $\partial/\partial p_A|_{p=0}$ to Eq. (5.14). We obtain

$$\frac{\partial}{\partial q_A} F(z|q, \tilde{r}) = \Psi^A(z|q+r) - \Psi^A(z|q), \quad (5.15)$$

$$\Psi^A(z|q) = \left. \frac{\partial}{\partial p_A} F(z|p, \tilde{q}) \right|_{p=0}.$$

It follows from Eq. (5.15)

$$\frac{\partial}{\partial q_A} \Psi^B(z|q+r) - (-1)^{\varepsilon_A \varepsilon_B} \frac{\partial}{\partial q_B} \Psi^A(z|q+r) = \frac{\partial}{\partial q_A} \Psi^B(z|q) - (-1)^{\varepsilon_A \varepsilon_B} \frac{\partial}{\partial q_B} \Psi^A(z|q) \quad (5.16)$$

and then

$$\frac{\partial}{\partial r_A} \Psi^B(z|r) - (-1)^{\varepsilon_A \varepsilon_B} \frac{\partial}{\partial r_B} \Psi^A(z|r) = -(-1)^{\varepsilon_A \varepsilon_B} \omega^{AB}(z), \quad (5.17)$$

$$\omega^{AB}(z) = \left. \frac{\partial}{\partial q_B} \Psi^A(z|q) - (-1)^{\varepsilon_A \varepsilon_B} \frac{\partial}{\partial q_A} \Psi^B(z|q) \right|_{q=0} = -(-1)^{\varepsilon_A \varepsilon_B} \omega^{BA}(z).$$

A general solution of Eq. (5.17) is

$$\Psi^A(z|r) = \frac{1}{2}\omega^{AB}(z)r_B + \frac{\partial}{\partial r_A}\phi(z|r). \quad (5.18)$$

Function (5.18) satisfies Eq. (5.16) also. Substituting Exp. (5.18) in Eq. (5.15), we obtain

$$\frac{\partial}{\partial q_A} \left(F(z|q, \tilde{r}) - \frac{1}{2}q_A\omega^{AB}(z)r_B - \phi(z|q+r) + \phi(z|q) \right) = 0$$

and as a result

$$F(z|q, r) = \frac{1}{2}q_A\omega^{AB}(z)\tilde{r}_B + \phi(z|q+\tilde{r}) - \phi(z|q) + \varphi(z|\tilde{r}). \quad (5.19)$$

Substituting Exp. (5.19) in Eq. (5.14) we find that function (5.19) satisfies Eq. (5.14) if function $\varphi(z|r)$ is equal to $\varphi(z|r) = -\phi(z|r) - \varphi_1(z)$. So, we obtain

$$F(z|p, q) = \frac{1}{2}p_A\omega^{AB}(z)\tilde{q}_B + \phi(z|p+\tilde{q}) - \phi(z|p) - \phi(z|\tilde{q}),$$

where a redefinition $\phi(z|p) \rightarrow \phi(z|p) + \varphi_1(z)$ was made, or

$$\begin{aligned} \Phi_{2\text{loc}}(z|f, g) &= \frac{1}{2}f(z)\overleftarrow{\partial}_A\omega^{AB}(z)\partial_B g(z) + d_1\Phi_{1|2}(z|f, g), \\ \Phi_{1|2}(z|f) &= -f(z)\sum_{k=0}^K(\overleftarrow{\partial}_A)^k\phi^{(A)_k}(z), \end{aligned}$$

where $\phi^{(A)_k}(z)$ are coefficients of the polynomial $\phi(z|p)$, $\phi(z|p) = \sum_{k=0}^K(p_A)^k\phi^{(A)_k}(z)$.

Finally, with (5.12) taken into account, we find that general solution of cohomological equation (5.1) has the form

$$\begin{aligned} \Phi_2(z|f, g) &= m_\omega(z|f, g) + d_1\Phi_1(z|f, g), \\ m_\omega(z|f, g) &= \frac{1}{2}f(z)\overleftarrow{\partial}_A\omega^{AB}(z)\partial_B g(z) = -(-1)^{\varepsilon(f)\varepsilon(g)}m_\omega(z|g, f), \quad \varepsilon(\omega^{AB}) = \varepsilon_A + \varepsilon_B, \\ \Phi_1(z|f) &= \Phi_{1|1}(z|f, g) + \Phi_{1|2}(z|f). \end{aligned} \quad (5.20)$$

The cohomologies with different $\omega^{AB}(z)$ are independent. Indeed, consider an equation

$$f(z)\overleftarrow{\partial}_A\omega^{AB}(z)\partial_B g(z) = d_1\Psi_1(z|f, g) = f(z)\Psi_1(z|g) - \Psi_1(z|fg) + \Psi_1(z|f)g(z). \quad (5.21)$$

Let

$$[z \cup \text{supp}(g)] \cap \text{supp}(f) = \emptyset.$$

We find from Eq. (5.21)

$$\hat{\Psi}_1(z|f) = 0,$$

i.e. the form $\Psi_1(z|f)$ is local:

$$\Psi_1(z|f) = f(z)\sum_{k=0}^K(\overleftarrow{\partial}_A)^k\psi^{(A)_k}(z) \equiv f(z)\psi(z|\overleftarrow{\partial}).$$

Let $f(z) = e^{pz}$, $g(z) = e^{qz}$. It follows from Eq. (5.21)

$$p_A \omega^{AB}(z)(-1)^{\varepsilon_B} q_B = \psi(z|q) - \psi(z|p+q) + \psi(z|p).$$

R.h.s. of this equation is symmetric under exchange $p \leftrightarrow q$, such that we have

$$p_A \omega^{AB}(z)(-1)^{\varepsilon_B} q_B = q_B \omega^{BA}(z)(-1)^{\varepsilon_A} p_A = -p_A \omega^{AB}(z)(-1)^{\varepsilon_B} q_B$$

from what it follows $\omega^{AB}(z) = 0$, that is, Eq. (5.21) has solutions only for $\omega^{AB}(z) = 0$.

References

- [1] *F. Bayen, M. Flato, C. Fronsdal, A. Lichnerovich and D. Sternheimer*, Ann.Phys., **111**, 61 (1978); Ann.Phys., **111**, 111 (1978).
- [2] *M. V. Karasev and V. P. Maslov*, Nonlinear Poisson brackets. Geometry and Quantization [in Russian], Nauka, Moscow (1991); *M. V. Karasev and V. P. Maslov*, Nonlinear Poisson brackets. Geometry and Quantization, AMS, Providence, RI (1993).
- [3] *B. Fedosov*, Deformation quantization and Index Theory, Akademie Verlag, Berlin, 1996.
- [4] *M. Kontsevich*, Deformation quantization of Poisson manifolds, I, q-alg/9709040.
- [5] *N. Nekrasov*, Trieste Lectures on solitons in noncommutative gauge theories, hep-th/0011095.
- [6] *A. Schwarz*, Gauge theories on noncommutative spaces, hep-th/0011261.
- [7] *H. Groenewold*, Physica, **12**, 405 (1946).
- [8] *F. A. Berezin*, Uspekhi Fiz. Nauk, **23** (1980).
- [9] *I. V. Tyutin*, The general form of the star-product on the Grassmann algebra, hep-th/0101046.
- [10] *D. A. Leites and I. M. Shchepochkina*, How to quantize the antibracket, Theor. Math. Phys., **126**, 281 (2001).
- [11] *S. E. Konstein, I. V. Tyutin*, General form of the deformation of Poisson superbracket on (2,2)-dimensional superspace, hep-th/061206.
- [12] *V. V. Zharinov*, Hochschild cohomologies of the algebra of smooth functions, Theor. Math. Phys., **140**, 1195 (2004).
- [13] *S. E. Konstein, A. G. Smirnov and I. V. Tyutin*, Cohomologies of the Poisson superalgebra, Theor. Math. Phys., **143**, 625 (2005); hep-th/0312109.
- [14] *V. V. Zharinov*, Theor. Math. Phys., **136**, 1049 (2003).